Framework comparison between a multifingered hand and a parallel manipulator

Júlia Borràs and Aaron M. Dollar

Abstract In this paper we apply the kineto-static mathematical models commonly used for robotic hands and for parallel manipulators to an example of hand-plus-object (parallel manipulator) with three fingers (legs), each with two phalanges (links). The obtained analytical matrix expressions that define the velocity and static equations in both frameworks are shown to be equivalent. This equivalence clarifies the role of the grasp matrix versus the parallel manipulator Jacobian. Potential knowledge transfer between both fields is discussed in the last section.

Key words: Parallel Manipulators, Multifingered Robotic Hands, Screw Theory.

1 Introduction

A hand manipulating an object held in the fingertips has the same kinematic structure as a parallel manipulator where the platform is the object and the legs are the fingers. Despite this fact has been acknowledged by many authors [10, 6], few works discuss connections between the mathematical frameworks of both systems [4]. A hand-plus-object system is a highly redundant hybrid parallel manipulator, where the only passive joints are the contact attachments. However, the hand-plus-object system has to hold an extra condition: the fingertip force has to be directed towards the object and inside the friction cone [12]. This condition does not modify the kineto-static mathematical model, because it is treated as a constraint when solving the static equations.

This paper reviews the mathematical frameworks involved for modeling the hand-plus-object system of a hand with three fingers and two phalanges per finger, and its kinematically equivalent parallel manipulator. As expected, we show

This work was supported in part by National Science Foundation grant IIS-0952856.
The authors are with the Department of Mechanical Engineering and Materials Science, Yale University, New Haven, CT USA. (e-mail: julia.borrassol, aaron.dollar@yale.edu).

how the derived static equations match. We believe that our comparison helps to clarify the role of the grasp matrix versus the role of the parallel manipulator Jacobian matrix. As far as the authors know, there has not been any publication proving that both frameworks are analytically equivalent. The results obtained in this paper are for a particular example. A general complete proof of such equivalence is left as future work.

Section 2 introduces the studied example and its notation. Section 2.1 details the steps to obtain the matrices for hands and Section 2.2 for parallel robots. The obtained matrices are compared in Section 2.3. Finally, Section 3 discusses advantages of the proven equivalence, and proposes future work based on transfer of knowledge between both fields.

2 The 3-UR hand and its equivalent parallel manipulator

This paper analyzes the three-fingered hand depicted in Fig. 1. Its architecture is similar to other robotic hands such as the Barrett hand [16] or the JPL hand [13]. The hand consists of three equal fingers with two phalanges each and three rotational joints each (2 in finger flexion, and one base rotation). For each finger i, $\mathbf{z}_{i1} = (0,0,1)^T$ and $\mathbf{z}_{i2} = \mathbf{z}_{i3} = (\sin(\theta_{i1}), -\cos(\theta_{i1}), 0)^T$ are the axis of rotation of the first, second and third joints, respectively, with rotation angles θ_{i1}, θ_{i2} and θ_{i3}, respectively (see Fig. 1).

To complete the hand-plus-object system, we need to define the contact model. The two most common contact models are called hard and soft fingers. The first one assumes a point contact with friction with a small contact patch. Kinematically, it is equivalent to a spherical joint. The second model assumes a larger contact patch and thus, the finger can also transmit a moment about the contact normal. This is equivalent to a universal joint. Therefore, the system hand-plus-object using the hard-finger (soft-finger) model is kinematically equivalent to a 3-URS (3-URU) parallel manipulator (where U stands for universal joint, R revolute, and S spherical). In this work, we use the hard-finger model. Then, the mobility of the manipulator, computed using the Grübler-Kutzbach criterion, is 6, that means the object (platform) can be moved in 6 degrees of freedom (DoF). Other more complex models, such as the rolling contact, are left as future work [15].

Hands need to actuate all the joints to keep the fingers rigid when they work without contact, and thus, the resulting manipulator will have the 9 finger joints

$$\Theta = (\theta_{11}, \theta_{12}, \theta_{13}, \theta_{21}, \theta_{22}, \theta_{23}, \theta_{31}, \theta_{32}, \theta_{33}) \tag{1}$$

actuated. The rest of the joints are left free to move (passive). They are defined considering the spherical joints as the intersection of three revolute joints. We define their axis of rotation as $\mathbf{z}_{i4} = (1,0,0)$, $\mathbf{z}_{i5} = (0,1,0)$ and $\mathbf{z}_{i6} = (0,0,1)$, with angles θ_{i4}, θ_{i5} and θ_{i6}, respectively. Then, we can state that the manipulator has three degrees of *actuation redundancy* (9 actuated joints versus 6 DoF of mobility). As the

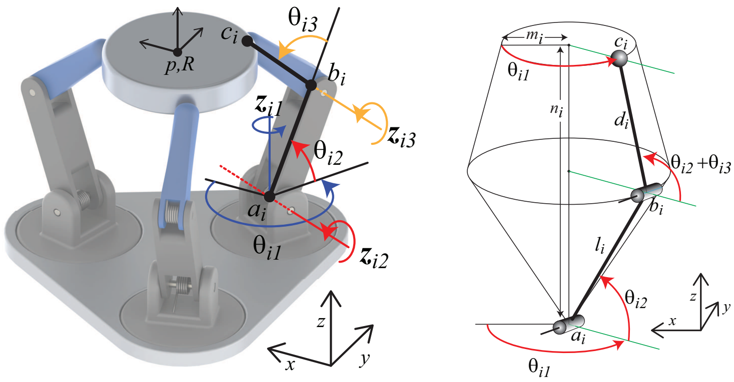


Fig. 1 A three-fingered hand with its corresponding notation. The center points of the palm joints are equally distributed around a circumference of radius r_p, and the contact points on the object around a circumference of radius r_o. By geometric construction, the coordinates of the fingertip can be described using the magnitudes $n_i = l_i \sin(\theta_{i2}) + d_i \sin(\theta_{i2}+\theta_{i3})$ and $m_i = l_i \cos(\theta_{i2}) + d_i \cos(\theta_{i2}+\theta_{i3})$, where l_i and d_i are the lengths of the proximal and distal links of the ith finger, respectively.

output twist that defines the velocity and angular velocity of the object(platform) is also 6 dimensional, we can say that the manipulator does not have *kinematic redundancy* [19].

The position and orientation of the object (platform) with respect to the palm (base) reference frame are given by a position vector $\mathbf{p} \in \mathbb{R}^3$ located at the center of mass of the platform (object) and a rotation matrix $\mathbf{R} \in SO(3)$. If $\widetilde{\mathbf{a}}_i$ and $\widetilde{\mathbf{c}}_i$ are the local coordinates of the palm (base) and object (platform) attachments in their local reference frames, their coordinates with respect to the palm (base) fixed reference frame are $\mathbf{a}_i = \widetilde{\mathbf{a}}_i$ and $\mathbf{c}_i = \mathbf{p} + \mathbf{R}\widetilde{\mathbf{c}}_i$. Assuming contact, the coordinates of the contact points must be the same as the coordinates of the fingertips, which can be obtained by geometric construction as

$$\mathbf{c}_i = \mathbf{a}_i + n_i(0,0,1)^T + m_i(\cos(\theta_{i1}),\ \sin(\theta_{i1}),\ 0)^T,$$

(where n_i and m_i are defined in Fig. 1-(right)). The loop equations are obtained equating the two obtained coordinates of the contact points $\mathbf{c}_i$. Solving them for Θ or for $\{\mathbf{p}, \mathbf{R}\}$ gives the the inverse and forward kinematic solutions, respectively.

The next two sections describe how to obtain the velocity equations using the grasping framework [12] and the parallel manipulators frameworks [9, 17]. The equations are listed in Table 1, for the described hand (first column of the table) and the equivalent parallel manipulator (second column of the table).

The velocity of the object (platform) is described using screw theory in both frameworks. We define a screw as $\$ = (\mathbf{u}, \mathbf{q} \times \mathbf{u})$ for a given vector $\mathbf{u}$ and a position vector $\mathbf{q}$. Two screws are reciprocal when its reciprocal product is zero, *i.e.*,

$$(\mathbf{u}_1, \mathbf{q}_1 \times \mathbf{u}_1) \circ (\mathbf{u}_2, \mathbf{q}_2 \times \mathbf{u}_2) = (\mathbf{q}_1 \times \mathbf{u}_1, \mathbf{u}_1) \cdot (\mathbf{u}_2, \mathbf{q}_2 \times \mathbf{u}_2) = 0,$$

where $\cdot$ stands for the usual dot product and $\circ$ the reciprocal product [3, 18]. The twist $\mathbf{T} = (\mathbf{v}, \Omega)$ defines the linear and angular velocity of the object (platform).

2.1 The grasp matrix and the hand Jacobian

The total grasp and hand Jacobian matrices are defined stacking together the matrices of each finger as shown in Table 1-row f. To define each finger matrix, first we need to define a set of reference frames, $\{C_i\} = \{\mathbf{c}_i, \mathbf{R}_i\}$, located at each of the contact points and with rotation matrix $\mathbf{R}_i = (\mathbf{n}_i\ \mathbf{t}_i\ \mathbf{o}_i)$, with $\mathbf{n}_i$ normal to the plane tangent to the object at the contact point, and directed toward the object. The remaining two vectors are chosen orthonormal to the first one (Table 1-row a). For our case, we define these vectors as

$$\begin{aligned} \mathbf{n}_i =& (n_{ix}, n_{iy}, n_{iz}) = \frac{\mathbf{p} - \mathbf{c}_i}{r_o}, \\ \mathbf{t}_i =& \left(\frac{n_{iy}}{\sqrt{n_{ix}^2 + n_{iy}^2}}, -\frac{n_{ix}}{\sqrt{n_{ix}^2 + n_{iy}^2}}, 0\right), \\ \mathbf{o}_i =& n_i \times t_i \end{aligned} \tag{2}$$

The grasp matrix for the finger i is a change of coordinates of the twist of the object $\mathbf{T}$, from the fixed reference frame to $\{C_i\}$. Let $\mathbf{T}_{fi}$ be the twist at the fingertip i with respect to the reference $\{C_i\}$. Then, $\mathbf{T}_{fi} = \mathbf{G}_i^T \mathbf{T}$ where $\mathbf{G}_i^T = \mathbf{H}_i \overline{\mathbf{R}}_i \mathbf{P}_i$ (see explicit expression in Table 1-row d). The matrix $\mathbf{P}_i$ translates the twist from $\mathbf{p}$ to $\mathbf{c}_i$. The matrix $\overline{\mathbf{R}}_i$ rotates the twist to match $\{C_i\}$ and $\mathbf{H}_i$ is the contact model matrix, that sets to zero the three coordinates corresponding to the angular velocity (see [12] for detailed definition of this matrix).

The hand Jacobian matrix $\mathbf{J}_H$ is defined by the joint twists, whose expressions for each finger i are

$$\begin{aligned} \$_{i1} =& ((\mathbf{a}_i - \mathbf{c}_i) \times \mathbf{z}_{i1}, \mathbf{z}_{i1})^T \\ \$_{i2} =& ((\mathbf{a}_i - \mathbf{c}_i) \times \mathbf{z}_{i2}, \mathbf{z}_{i2})^T \\ \$_{i3} =& ((\mathbf{b}_i - \mathbf{c}_i) \times \mathbf{z}_{i3}, \mathbf{z}_{i3})^T. \end{aligned} \tag{3}$$

Note that the angular components are computed about the center of the reference $\{C_i\}$. Then, the ith fingertip twist is expressed as $\mathbf{T}_{fi} = \mathbf{J}_{Hi}\dot{\Theta}$, where $\mathbf{J}_{Hi}$ is detailed in Table 1-row e. As before, the matrix $\overline{\mathbf{R}}_i$ is used to write the twist with respect to $\{C_i\}$ and $\mathbf{H}_i$ to select only the transmitted components.

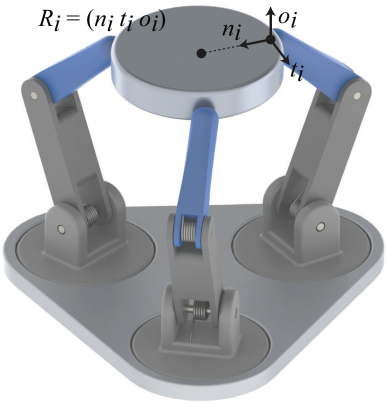
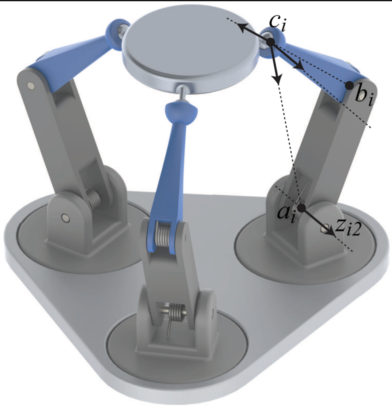
Finally, rows b and c show the velocity and the static equilibrium equations using the complete matrices.

2.2 The Jacobian matrix of the parallel manipulator

Here we follow the steps proposed in [9] or Chapter 5.6 in [17] to obtain the Jacobian matrix for the parallel manipulator shown in Table 1, row a, second column.

Let $\mathbf{T}$ be the twist of the platform, as in the previous section. The theorem in [9] states that it can be written as the sum of the joint twists of each leg, that is,

Table 1 Summary of static and velocity equations. τ and $\dot{\Theta}$ are the vector of joint torques and velocities, respectively. $\mathbf{W}$ and $\mathbf{T}$ are the external wrench and twist acting on the object(platform). λ_f is a 1×9 vector containing the three fingertip forces, and $\mathscr{F}$ represents the friction cone.

	Grasping	Parallel manipulators
a		
b	$\mathbf{G}^T\mathbf{T} = \mathbf{J}_H\dot{\Theta}$	$\mathbf{J_p}\mathbf{T} = \mathbf{J}_\Theta\dot{\Theta}$
c	$\mathbf{J}_H^T\lambda_f = \tau$ $-\mathbf{G}\lambda_f = \mathbf{W}$ $\lambda_f \in \mathscr{F}$	$\mathbf{W} = -\mathbf{J_p}^T\mathbf{J}_\Theta^{-T}\tau$
d	$\mathbf{G}_i^T = \mathbf{H}_i\begin{pmatrix}\mathbf{R}_i & 0\\ 0 & \mathbf{R}_i\end{pmatrix}\begin{pmatrix}\mathbf{I}_3 & 0\\ (\mathbf{c}_i-\mathbf{p})^\times & \mathbf{I}_3\end{pmatrix}$ where $\mathbf{v}^\times$ is the cross-product matrix	$\mathbf{J}_{pi} = \begin{pmatrix}\mathbf{z}_{i2}^T & (\mathbf{c}_i\times\mathbf{z}_{i2})^T\\ (\mathbf{c}_i-\mathbf{b}_i)^T & (\mathbf{c}_i\times(\mathbf{c}_i-\mathbf{b}_i))^T\\ (\mathbf{c}_i-\mathbf{a}_i)^T & (\mathbf{c}_i\times(\mathbf{c}_i-\mathbf{a}_i))^T\end{pmatrix}$
e	$\mathbf{J}_{Hi} = \mathbf{H}_i\overline{\mathbf{R}}_i(\$_{i1}\$_{i2}\$_{i3})$ with $\$_{ij}$ defined in (3)	$\mathbf{J}_{\Theta i} = \begin{pmatrix}-m_i & 0 & 0\\ 0 & l_id_iSin(\theta_{i3}) & 0\\ 0 & 0 & -l_id_iSin(\theta_{i3})\end{pmatrix}$ with m_i defined in Fig. 1
f	$\mathbf{G}^T = \begin{pmatrix}\mathbf{G}_1^T\\ \mathbf{G}_2^T\\ \mathbf{G}_3^T\end{pmatrix}$, $\mathbf{J}_H = \begin{pmatrix}\mathbf{J}_{H1} & 0 & 0\\ 0 & \mathbf{J}_{H2} & 0\\ 0 & 0 & \mathbf{J}_{H3}\end{pmatrix}$	$\mathbf{J_p} = \begin{pmatrix}\mathbf{J}_{p1}\\ \mathbf{J}_{p2}\\ \mathbf{J}_{p3}\end{pmatrix}$, $\mathbf{J}_\Theta = \begin{pmatrix}\mathbf{J}_{\Theta 1} & 0 & 0\\ 0 & \mathbf{J}_{\Theta 2} & 0\\ 0 & 0 & \mathbf{J}_{\Theta 3}\end{pmatrix}$

$$\mathbf{T} = \sum_{j=1}^{6} \dot{\theta}_{ij} \$_{ij}, \text{ for } i = 1,2,3. \tag{4}$$

Here, the joint twists angular components are computed about the center of the base fixed reference frame, namely, $\$_{ij} = (\mathbf{r} \times \mathbf{z}_{ij}, \mathbf{z}_{ij})$, for $j = 1,\dots,6$, where $\mathbf{r}$ takes the value of the corresponding joint center. The first three joint twists are equivalent to the twists defined in (3). The remaining three correspond to the passive joints.

The passive joint twists $\$_{ij}$, for $j = 4,5,6$, can be eliminated from the system (4) computing their reciprocal screws, named as ${}^{r}\$_{ik}$, for $k = 1,2,3$. It is important to note that any set of three linearly independent screws through the contact point are reciprocal to the spherical joint system $\{\$_{i4}, \$_{i5}, \$_{i6}\}$ [3, 18]. After multiplying the reciprocal system at both sides of each equation in (4), we can rewrite the system as $\mathbf{J_p T} = \mathbf{J}_{\Theta} \dot{\Theta}$, where the rows of the matrix $\mathbf{J_p}$ are the reciprocal screws and the matrix $\mathbf{J}_{\Theta}$ only depends on the active joint angles, $\mathbf{J}_{\Theta} = ({}^{r}\$_{ik} \circ \$_{ij})$, for $j = 1,\dots,3$ and $k = 1,\dots,3$. That is, it is formed by all the products of the reciprocal screws with the actuated joint screws.

The most convenient choice of the reciprocal screws is to define each one to be reciprocal to all the passive joint twists plus two of the active. This leads to a diagonal matrix $\mathbf{J}_{\Theta i}$ (Table 1-row e). The explicit expressions of the reciprocal screws for each leg i are the rows of the matrix $\mathbf{J}_{pi}$ in Table 1-row d.

We can obtain the ith fingertip wrench, written with respect to the fixed reference frame, by multiplying each set of three columns in $\mathbf{J_p}^T \mathbf{J}_{\Theta}^{-T}$ by the corresponding three joint torques τ. When the matrix $\mathbf{J}_{\Theta}$ is not square, we can use the pseudo-inverse.

2.3 Comparison of frameworks

We computed all the equations using Wolfram Mathematica 9. We can see that the matrices in the rows d, e and f between the two columns of Table 1 are obviously different. However, the analytical expression of the products $\mathbf{J}_{\Theta}^{-1}\mathbf{J_p}$ and $\mathbf{J}_H^{-1}\mathbf{G}^T$ are the same, except for the angular components. In the grasping framework, the angular velocities (moments) components of the twists (wrenches) are computed with respect to the center of the object (platform), while in the parallel manipulators framework they are computed with respect to the fixed reference frame center. Thus, we can say that they are equivalent $\mathbf{J}_{\Theta}^{-1}\mathbf{J_p} \equiv \mathbf{J}_H^{-1}\mathbf{G}^T$.

In section 2.2 we state that the reciprocal screws can be chosen arbitrarily, provided that they are independent and through the contact point $\mathbf{c}_i$. Then, let us define them using the vectors of the fingertip frame (see equation (2) and figure in Table 1, row a, column 1). In other words, we use $\mathbf{n}_i$, $\mathbf{t}_i$ and $\mathbf{o}_i$ to define the screws in the matrix $\mathbf{J}_{pi}$. Then, all the matrices in both frameworks coincide, that is, $\mathbf{J}_H = \mathbf{J}_{\Theta}$ and $\mathbf{J_p} \equiv \mathbf{G}^T$, where the second equivalence is not analytically identical only because the moments and angular velocities are computed with respect to different centers.

Note that the particular choice of the reciprocal screws will shape the final form of the matrices in Table 1. Analogously, in the grasping framework, this choice is made when defining the vectors of the rotation matrix of the reference frames $\{C_i\}$.

In the grasping context, the choice of the vectors $\{\mathbf{n}_i, \mathbf{t}_i, \mathbf{o}_i\}$ is convenient to obtain the expression of the fingertip forces λ_f directly projected to the axes of the friction cones. This facilitates the evaluation of the friction cone conditions. In the parallel manipulator context, the choice is done so that the resulting matrix $\mathbf{J}_\Theta$ is as diagonal as possible. This allows the interpretation of the rows of the complete Jacobian $\mathbf{J}_\Theta^{-1}\mathbf{J_p}$ in terms of line Plücker coordinates [7]. This is useful to find geometrical interpretation of singularities. Recently, in [2] they have used this technique to hand fingers, and the reciprocal system is chosen to facilitate the single value decomposition of the resulting finger Jacobian matrix.

We can also observe that the steps shown in Sections 2.1 and 2.2 can be generalized to any type of hand (manipulator), but the resulting matrices will be tall, wide or square depending on the relationship between the mobility, the number of actuated and passive joints and the dimension of the output twist [19]. It remains to proof that the results are always equivalent.

3 Discussion and future work

The grasping literature commonly uses the manipulability index to state the quality of the grasp, and it is either based only on the hand Jacobian [13] or on the multiplication of both matrices $\mathbf{J}_H^{-1}\mathbf{G}^T$ [14]. While this can detect singularities, the literature of parallel robots has extensively studied and classified them in much more detail [19, 5, 20].

Among parallel robot designers, it is well known that a smart design has to take into account the singularities inside the workspace [8, 1]. As far as the authors know, this is not done when designing hands. In part, this may be because the actuation redundancy reduces the dimensionality of the singularity locus. However, simplified hands that use underactuated fingers can reduce the degree of actuation redundancy down to 0 or even lower. In particular, we are studying how underactuation with pulling cables can be modeled with similar Jacobian matrices where these kind of singularities need to be taken into account. This type of hands are becoming very popular not only for effective grasps, but also to perform dexterous manipulation [11]. For these hands, singularities may be an issue that researchers will have to take into account in the process of hand design.

We believe that the study of convenient choices of the reciprocal system can lead to useful tools to design hands with increased workspaces. For instance, it can be useful to compute an analytical expression of the hyper-surface of singularities using only task space variables. Analyzing such surface can help to plot independent components inside a workspace, that cannot be crossed without loosing control.

This work has shown how the grasp matrix plays the same role as the Jacobian of reciprocal screws for the analyzed example. Such equivalence allows for transfer of knowledge from parallel manipulators to robotic hands. Extending this work to more general cases is part of a future work that will help to fully understand the parallelisms between these two types of manipulators.

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